Music Notation Font Design:
Technical Specifications for Note Head Font Design

Originally published © 1999 Sunhawk Corporation (now Sunhawk Digital Music LLC)
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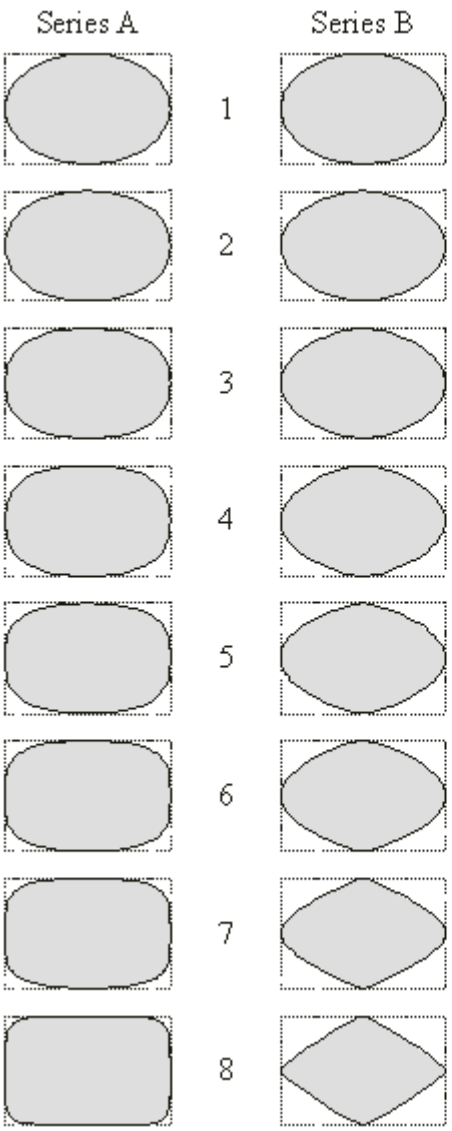
As a result of this paper and engraving samples that accompanied it, Sunhawk's music notation font "Sinfonia" was accepted as meeting the MPA standard according to the minutes of the Music Publisher's Association Board Meeting, February 3, 1999. The Sinfonia font, used in Sunhawk's Solero® Music Editing software, was designed and created by Gary Martin in the early part of 1995.

1.1 QUARTER NOTE (CROTCHET, VIERTELNOTE)

1.1.1 Elliptical Shape

1.1.1.1 *Ellipse and Modifications to an Ellipse*

The regular ellipse is the design shape of choice for regular quarter note heads. Font and graphics programs make it possible to modify the regular ellipse in numerous ways. A few examples are shown below. The only departure from the regular ellipse that might come under consideration for note head design would be Example 2 of Series A, with a slight elongation of the horizontal nodes.



MODIFICATIONS TO AN ELLIPSE

Example 1 of both Series A and Series B to the left are regular ellipses with a ratio of major to minor axes of 3:2. All modifications shown in Examples 2 through 8 are made on this ellipse.

Series A

In Examples 2 through 6 of Series A the horizontal nodes are stretched. The effect is to fill up the corners of the bounding rectangle.

In Examples 7 and 8 the vertical nodes are also stretched, and the figure approximates the shape of the rectangle.

Series B

In Examples 2 through 6 of Series B the horizontal nodes are reduced, giving more space to the corners of the bounding rectangle.

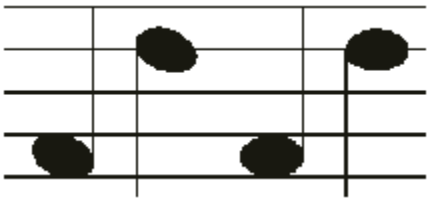
In Examples 7 and 8 the vertical nodes are also reduced, and the shape approximates a diamond.

In each case the modifications made are symmetrical about the axes. Non-symmetrical shapes do not fall under consideration for note head design purposes.

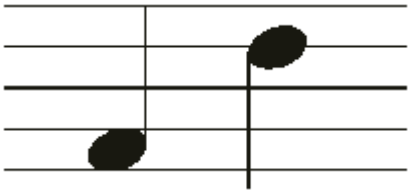
None of the modifications in Series B will fall under consideration for note head design since the surface area is reduced.

1.1.1.2. *Angle of Slant: Non-Standard Examples*

In some older music one occasionally finds note heads slanted with the long axis at a negative angle with respect to the staff lines (below left), or with the long axis horizontal or parallel with the staff lines (below right), but these are generally felt to be of inferior aesthetic quality.



1.1.1.3. *Angle of Slant: Standard Practice*



1.1.2. DESIGN PARAMETERS

There are four mathematical values that play a role in note head design*: (1) height,(2) width, (3) angle, (4) eccentricity,* or the relationship of long axis to short axis. Below are examples of varying the values in different combinations:

1.1.2.1. *Note heads with identical angles and eccentricities, but with varying heights and widths:*

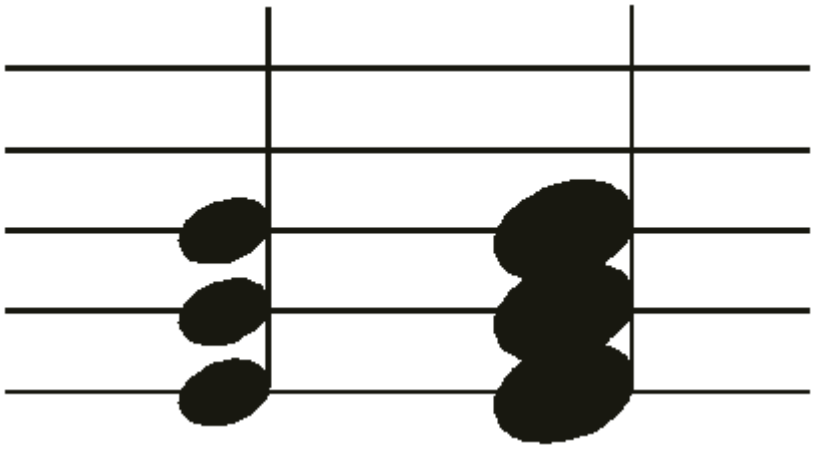
1.1.2.2. *Note heads with identical heights and eccentricities, but with varying angles and widths:*

1.1.2.3. *Note heads with identical heights and angles, but with varying eccentricities and widths:*

1.1.2.4. Depending on the constraints we put on the note head design, these four values will depend upon each other in different ways. We begin by addressing the first primary constraint — *height.*

1.1.3. HEIGHT

1.1.3.1. Any height that is less than the distance between two staff lines will result in a spatial separation of notes in a chord and a size that is simply too small, except for use as cues or graces (below left). Any height that is greater than the distance between two staff lines will result in excessive overlapping of notes in a chord, yielding an unclear or blurred effect (below right).



The height of a note head, regardless of its angle or eccentricity, should thus be constrained to the distance between two staff lines.

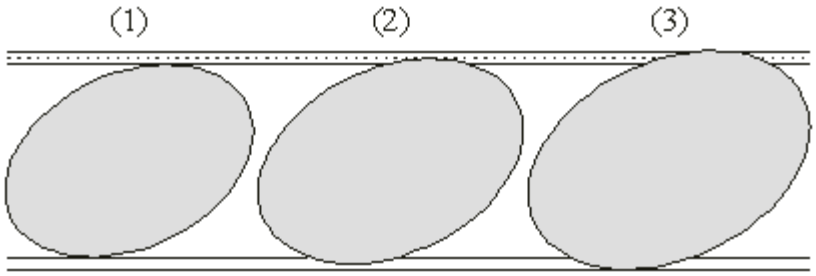
1.1.3.2. *Effect of Staff Line Thickness in Height Determination.*

As insignificant as it may seem, the staff line thickness itself has to be taken into consideration in the precise determination of the height. One may consider three options for setting the height of a note head between two staff lines:

1.1.3.2.1. Top of note head touching the bottom of a staff line above it,
Bottom of note head touching the top of a staff line below it—(1) below.

1.1.3.2.2 Top of note head reaching to center of a staff line above it,
Bottom of note head reaching to center of a staff line below it—(2) below.

1.1.3.2.3. Top of note head reaching to top of a staff line above it,
Bottom of note head reaching to bottom of a staff line below it—(3) below.



If we allow proportional increase of width as we increase height, the increase in overall size (surface area) increases significantly. In fact, in the particular case illustrated, the surface area of ellipse (2) is ca. 13% greater than that of ellipse (1), and the surface area of ellipse (3) is over 28% greater than that of ellipse (1). If we constrain width and increase height only, the difference between (3) and (1) is still significant (slightly over 8% greater surface area).

1.1.3.3. *Staff Line Thickness Relative to Staff Line Spacing.*

In the illustration above, the staff line thickness and staff line spacing are shown in the ratio used in the Solero® Editor (1:16). Different ratios will lessen or exaggerate the surface area differences shown.

1.1.3.4. *Overlap of Note Heads in a Stacked Chord.*

If we include staff line thickness in the height parameter of note head design, there may occur some small overlap of note heads in stacked chords, depending on angle and eccentricity parameters. However, the overlap zone is contained entirely within the staff line thickness and does not cause any significant blurring effect.

1.1.3.5. HEIGHT RULE

In the interest of keeping note head sizes as large as possible, while avoiding extension beyond staff line thickness zones, we arrive at the following rule for note head height:

The height of a note head shall be the distance between two staff lines including the staff line thickness.

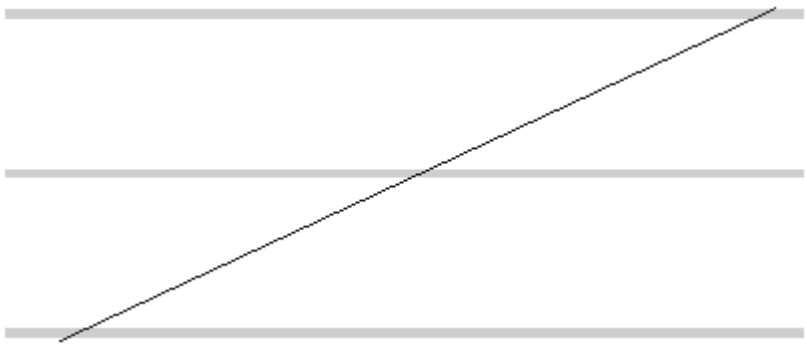
1.1.4. ANGLE AND ECCENTRICITY: DESIGN OPTION 1

1.1.4.1. A possible design option for setting the angle of note heads takes as its starting point the graphical relationship of note heads a second apart in pitch on the same stem. The rule, which really consists of two parts, is expressed as follows:

The long axes of seconds shall share the same line. The seconds shall touch each other but not overlap.

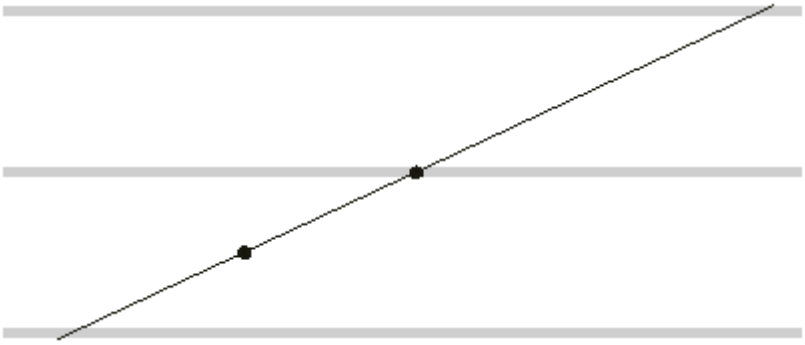
1.1.4.2. It is of interest to note that if these two conditions are accepted, angle and eccentricity will be mathematically dependent on each other—stipulate the one, and the other is automatically determined. If you decide you want to see the note heads slanted at a particular angle, you will thereby have defined the major and minor axes of the note head. If on the other hand you decide you want to see a particular relationship of the axes, you will thereby have defined the angle. Furthermore, the width is also determined. It remains then only to find a suitable pair of angle/eccentricity values, and the note head design is finished.

1.1.4.3. To understand the effect of the two constraints stated above on note head design, let's begin by drawing a line at some arbitrary angle on a staff and see how choice of angle determines the shape of the note head, given the fact that we have also already established a height rule. The shaded lines represent the staff lines.

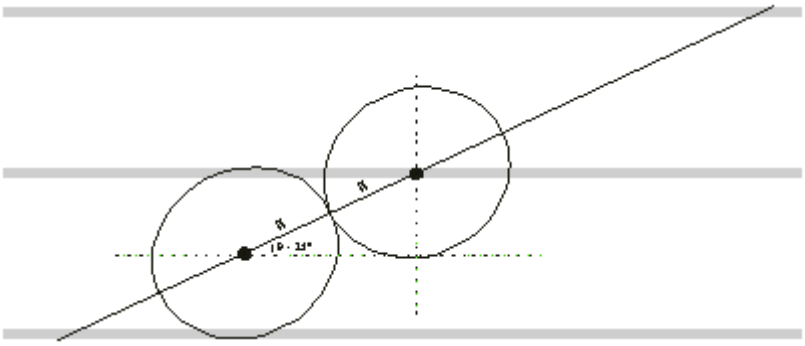


1.1.4.4. The line shown above will be the line common to the major axes of our note heads to be placed a second apart. By reason of vertical symmetry, the centers of the note heads must be located either centered in the thickness a staff line, or exactly half way

between the centers of two staff lines. This gives us two points we can now put on the line, which will be the centers of our note heads.



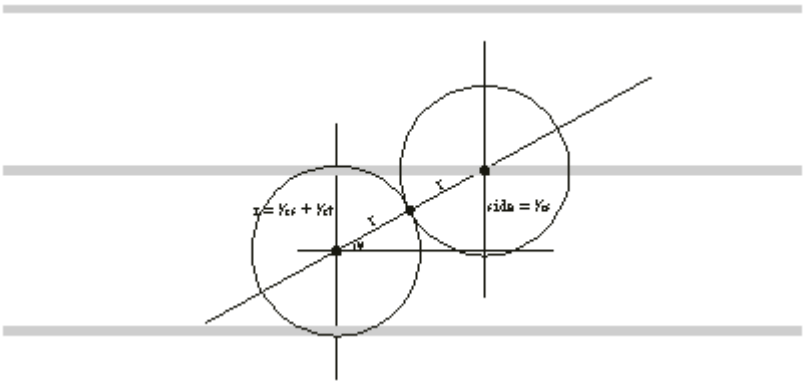
1.1.4.5. Imagine a point on the line exactly half way between the two points shown. That half-way point will be where the two note heads meet. Therefore the distance between either point and the half-way point equals the length of the semi-major axis of the note heads. Let's draw in the note heads to see this more clearly, and add a dotted horizontal line passing through the center of the lower note head as well as a vertical line passing through the center of the second note head.



1.1.4.6. Since the vertical distance between the centers of the two note heads is precisely known (half the distance between the centers of the staff lines), the length of the semi-major axis can be calculated: $\sin q = 0.5s/2a$ where s is the vertical distance between the centers of two staff lines. The minor axis is now also constrained to a unique value that results in the top-most and bottom-most points of the ellipse touching the preset vertical boundaries established by the height rule. Thus, *angle determines eccentricity.*

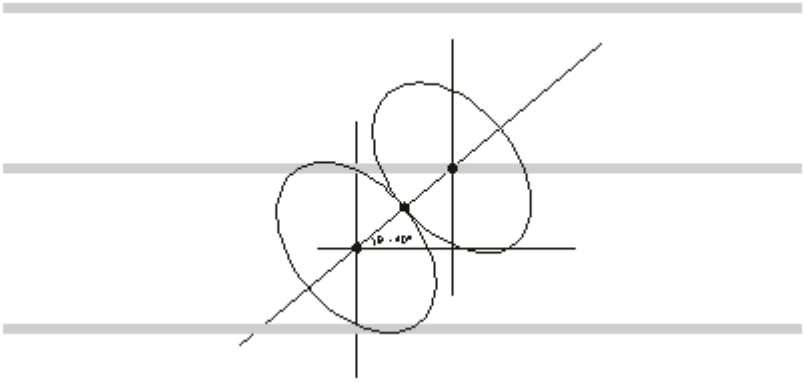
1.1.4.7. The general behavior of note head shapes as the angle changes is illustrated in the following examples.

1.1.4.7.1. Consider first the case in which the major and minor axes of an ellipse equal each other (a = b) resulting in a circle (eccentricity = 0).

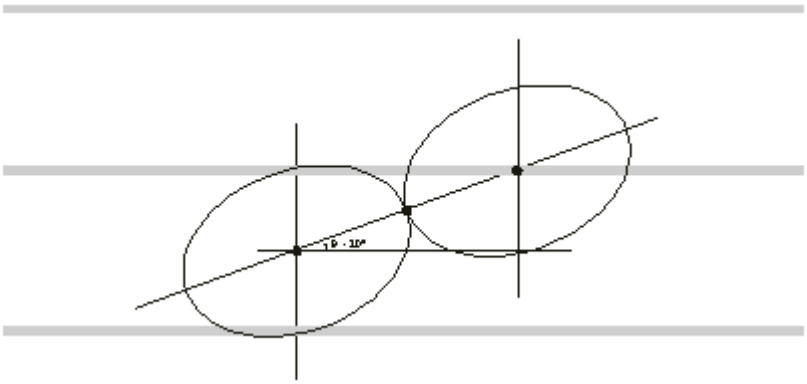


Let s = the distance between the *centers* of two staff lines, and t = the staff line thickness. The diameter of the circle = s+t and the radius = ½(s+t). In the triangle shown, the hypotenuse equals 2r, and the vertical side equals half of the distance between two staff lines (½s). Solving for the angle [$\sin \varphi = (½s)/2r = s/2(s+t)$] and using Solero® Editor units of s = 512 and t = 32, angle $\varphi = 28°$. [In a notation system where the note heads reach only to the centers of the staff lines, $\theta = 30°$, since the thickness factor (t) falls out and the equation is then simply: $\sin \theta = s/2s = 0.5$.]

1.1.4.7.2. Any angle above 28° will create note heads with negative angles. Below shows what happens when the angle is increased to 40°.

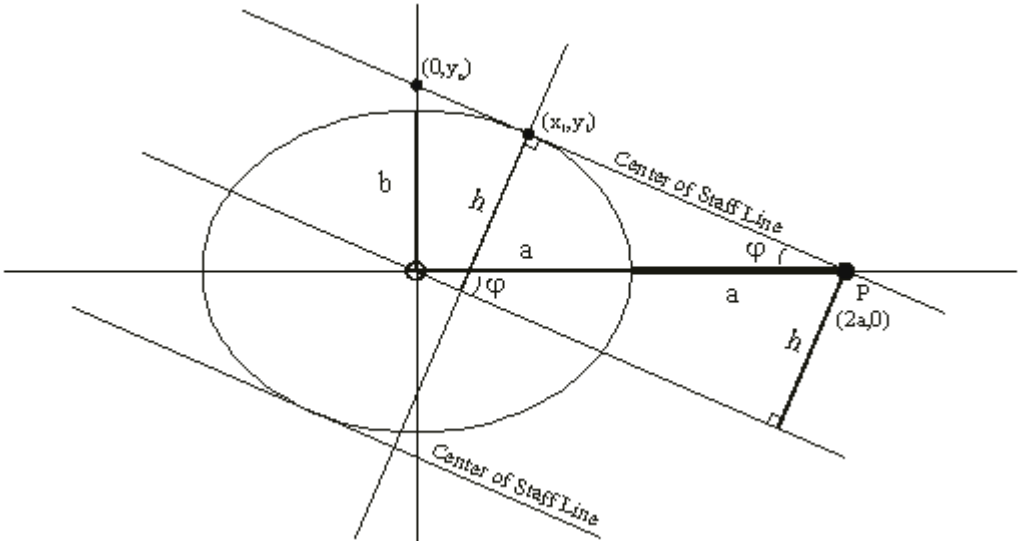


1.1.4.7.3. When the angle is kept below 28° the note heads will slant with a positive angle. Below is the case when the angle equals 20°.



1.1.4.7.4. It remains only to find the angle/eccentricity combination that seems most aesthetically pleasing. All the values can be determined precisely using basic mathematical principles of geometry, which are shown next, first for the condition not including staff line thickness, then modifying for it.

1.1.4.8. First we rotate the ellipse so that the major axis is horizontal and becomes the x-axis. The known quantities are *h* (a preset value which equals half the distance between the centers of two staff lines), and angle φ, the preset angle. We have a line tangent to the ellipse at point (x_1,y_1) intersecting the x-axis at point P(2a,0). We can solve for values of *a* (semi-major axis) and *b* (semi-minor axis).



1.1.4.9. Semi-Major Axis, *a*. The length 2a equals the length of the hypotenuse of the triangle with short side *h*.
Thus:

$$\sin\varphi = \frac{h}{2a}$$

$$a = \frac{h}{2\sin\varphi}$$

1.1.4.10. Semi-Major Axis, *b*. We start by constructing two equations for the tangent line. (1) The general equation for the line tangent to the point (x_1,y_1) on the ellipse is:

$$\frac{x_1 x}{a^2} + \frac{y_1 y}{b^2} = 1$$

At point P(2a,0) we have:

$$\frac{x_1 2a}{a^2} = 1$$

$$x_1 = \frac{a}{2} = \frac{h}{4\sin\varphi}$$

(2) Another equation for the tangent line is constructed by finding the slope and the y-intersect.

Slope:

$$-\tan\varphi$$

y intersect (y_o):

$$y_o = 2a\tan\varphi$$

So the equation of the line is:

$$y = -(\tan\varphi)x + 2a\tan\varphi = (2a - x)\tan\varphi$$

At point (x_1,y_1), substituting for x_1 from equation (1):

$$y_1 = \left(2a - \frac{a}{2}\right)\tan\varphi = \frac{3a\tan\varphi}{2}$$

or in terms of *h* using

$$a = \frac{h}{2\sin\varphi}$$

$$y_1 = \frac{3h\tan\varphi}{4\sin\varphi} = \frac{3h}{4\cos\varphi}$$

Now substitute the values for x_1 from equation (1) and y_1 from equation (2) into the equation for the ellipse. First solve for b in the general equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b = \frac{y}{\sqrt{1 - x^2/a^2}}$$

Then substitute and simplify:

$$b = \frac{3h/4\cos\varphi}{\sqrt{1 - \frac{(a^2/4)}{a^2}}} = \frac{3h/4\cos\varphi}{\sqrt{1 - \frac{1}{4}}} = \frac{3h/4\cos\varphi}{\sqrt{3}/2} = \frac{\sqrt{3}h}{2\cos\varphi}$$

1.1.4.11. Summary. The equations for the semi-major and semi-minor axes are:

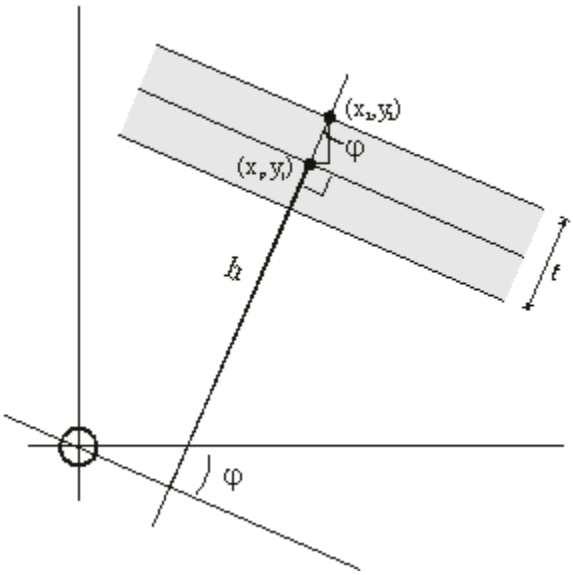
$$a = \frac{h}{2\sin\varphi}$$

$$b = \frac{\sqrt{3}h}{2\cos\varphi}$$

where the values *h* and j are given.

1.1.4.12. Conclusion of this Section.
In a given notation system where the distance between the centers of two staff lines has been set, the choice of angle of the shared major axes for the note heads of seconds automatically determines the values for major and minor axes (and therefore also the width as we will see).

1.1.4.13. Modifying to Include Staff Line Thickness in Height of Note Head.
We want to include the staff line thickness in the overall height of the note head, but not adjust the length of the major axis. What effect does increasing *h* have on *b*? In the illustration shown below, the staff line is represented by the shaded zone, which has been enlarged to show the detail.



We want to increase *h* by the length ½*t*. The new values for the point (x_2,y_2) in terms of (x_1,y_1) are:

$$x_2 = x_1 + \frac{t}{2}\sin\varphi$$

$$y_2 = y_1 + \frac{t}{2}\cos\varphi$$

Using earlier expressions for x_1 and y_1, we have:

$$x_2 = \frac{h}{4\sin\varphi} + \frac{t}{2}\sin\varphi$$

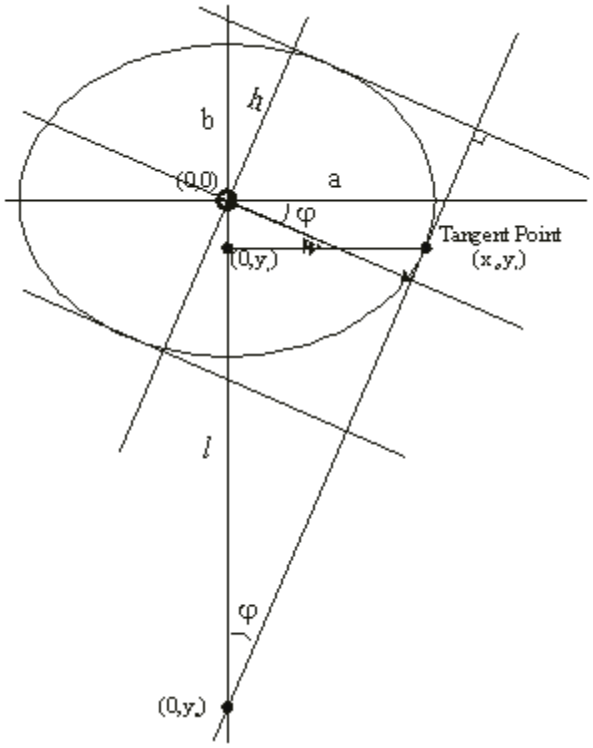
$$y_2 = \frac{3h}{4\cos\varphi} + \frac{t}{2}\cos\varphi$$

Using known values for *h*, *t*, and φ, calculate x_2 and y_2, then solve for *b* using the standard ellipse equation. The final equation for *b* in terms of known quantities is:

$$b = \frac{3h^2 + 2ht\cos^2\varphi}{2\cos\varphi\sqrt{4h^2 - (h + 2t\sin^2\varphi)^2}}$$

which reduces to the earlier equation for *b* when $t = 0$ (see 1.1.5.11).

1.1.4.14. Width.
To find the width, consider first the diagram shown below:



One equation for the line passing through the Tangent Point is [see 1.1.5.10 (1)]:

$$y = -\left(\frac{x_1 b^2}{y_1 a^2}\right)x + \frac{b^2}{y_1}$$

which results in the y-intercept value of:

$$y_o = \frac{b^2}{y_1}$$

Now,

$$\sin\varphi = \frac{w}{y_o}$$

so,

$$w = y_o \sin\varphi = \frac{b^2 \sin\varphi}{y_1}$$

Another equation for the same line is found by using the slope/intersect method [see 1.1.5.10 (2)]:

Slope:

$$\frac{1}{\tan\varphi}$$

so,

$$\frac{1}{\tan\varphi} = -\left(\frac{x_1 b^2}{y_1 a^2}\right)$$

since the slopes from the two equations are equal.

Thus,

$$y_1 = -\frac{x_1 b^2 \tan\varphi}{a^2}$$

To find the y-intercept, let *l* be the length from y_1 to y_o.

$$\tan\varphi = \frac{x_1}{l}$$

$$l = \frac{x_1}{\tan\varphi}$$

So the y-intercept is at:

$$y_o = y_1 - l = y_1 - \frac{x_1}{\tan\varphi}$$

(y_1 is negative)

Since the y-intercepts from the two equations are also equal, we find that:

$$\frac{b^2}{y_1} = \left(y_1 - \frac{x_1}{\tan\varphi}\right)$$

$$x_1 = \frac{\left(y_1^2 - b^2\right)\tan\varphi}{y_1}$$

Substituting this expression for x_1 into the equation for y_1 above, we obtain:

$$y_1 = \frac{b^2\tan\varphi}{a\sqrt{1 + \frac{b^2\tan^2\varphi}{a^2}}}$$

Now we can solve for *w* substituting this expression for y_1 in

$$w = \frac{b^2\sin\varphi}{y_1}$$

which results in:

$$w = a\cos\varphi\sqrt{1 + \frac{b^2\tan^2\varphi}{a^2}}$$

Note that if $\varphi = 0°$, $w = a$.
(Empirical tests show also that as φ approaches 90°, *w* approaches *b*.)

1.1.4.15. Table of Values.

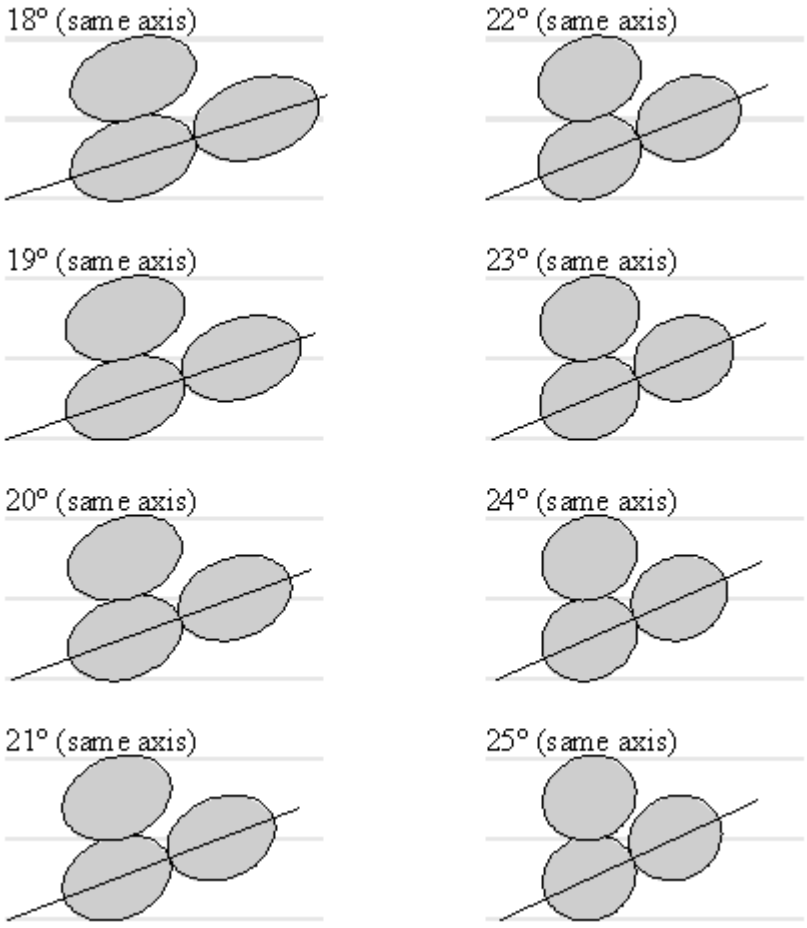
Based on experimental tests, the practical range of angles for note heads will lie between 18° and 24°. Using the equations derived in the preceding sections, the following table of

values has been generated. (Separation is the horizontal distance between centers of note heads). Units are Solero® Editor units.

See the end of this document for print samples for each note head angle listed below.

Angle	Major Axis	Minor Axis	Width	Separation
18°	**828**	**505**	**803**	**788**
19°	**786**	**508**	**762**	**743**
20°	**748**	**512**	**725**	**703**
21°	**714**	**515**	**692**	**667**
22°	**683**	**519**	**663**	**634**
23°	**655**	**523**	**637**	**603**
24°	**629**	**527**	**614**	**575**
25°	**606**	**531**	**593**	**549**

1.1.4.16. Graphical Samples (created from Table of Values).
The samples below show the alignment of seconds and the effect of stacked notes in a chord for angles from 18° to 25°. Note the tiny overlap of the note heads in the stacked chords, as mentioned in 1.1.4.4 (confined to thickness zone).



1.1.4.17. Construction of Note Heads in Font or Graphic Software.
Using the Table of Values, it is an easy matter to construct note heads that will precisely meet all the criteria addressed in “Option 1” of note head design. Here is the procedure for creating the note head with angle of 20°.

1.1.4.17.1. Enter a circle of diameter equal to the minor axis (=512).
1.1.4.17.2. Scale the circle horizontally only by a factor of 748/512 (=146.09%).
1.1.4.17.3. Rotate the scaled circle, now an ellipse, by +20°.
1.1.4.17.4. Set the position in the em-square according to font design specifications.

1.1.4.18. Engraving Samples. [Not included here.]

1.1.4.19. Evaluation.
Based on the graphical samples above and the attached engraving samples, one can eliminate at least the 18° sample as too wide and the 25° as too rounded for good note head design. The 21° and 22° samples seem match the older plate engraving sample provided by MPA the best.

1.2. HALF NOTE (MINIM, HALBENOTE)

1.2.1. Design Relationship To Quarter Note

1.2.1.1. For consistency in appearance, the general design criteria adopted for the quarter note head should be applied to the half note head. For example, if quarter note heads are to be aligned with their major axes along the same line, half note heads should follow the same pattern at the same angle.

1.2.1.2. The only new design aspect of half note heads is the shape and size of the counter (the enclosed white space inside the half note head).

1.2.2. Half Note Head Counter Design

1.2.2.1. The half note head is not merely a “hollow” version of the quarter note head. The counter is not just a smaller version of the half note head, as shown below.

Not this:

1.2.2.2. The counter should be proportionally narrow at the minor axes than at the major axes, as shown below.

1.2.2.3. Counter Values.

1.2.2.3.1. Major Axis of counter is ca. 85% that of the major axis of the note head.
1.2.2.3.2. Minor Axis of counter is ca. 45% that of the minor axis of the note head.

1.2.2.3.3. The vertical node separation of the counter is increased to 154% of the ellipse vertical node separation. The half note head counter is simply a vertical node stretch on an ellipse with a higher eccentricity than the ellipse used for the illustration there.

1.2.2.4. The values listed above should not be viewed as absolute. Certainly a range about these values will yield acceptable results.

1.3 WHOLE NOTE (SEMIBREVE, GANZENOTE)

1.3.1. General Design Aspects

1.3.1.1. The whole note head design is complex, which has given rise to perhaps a wider variety of designs among music font designers than is the case with the quarter and half note heads.

1.3.1.2. The general design parameters include:

1.3.1.2.1. Ratio of width to height.
1.3.1.2.2. Shape of outer perimeter.
1.3.1.2.3. Shape and size of counter.

1.3.2. Whole Note Head Width/Height Ratio

1.3.2.1. The width/height ratio for the whole note head used in the Solero® Editor was decided upon after evaluating whole notes from a variety of music publishers. Some publishers use wider whole notes, others use narrower ones. A compromise position was struck for the Sinfonia font.

1.3.2.2. The ratio for the whole note head in Sinfonia is: 1.957 (1002 units wide by 512 units high). Unlike the quarter and half note heads, the height of the whole note head was kept at the distance between the *centers* of two staff lines to avoid an extensive vertical overlap zone along the horizontal that would be created due to the flatter curves at the top and bottom of the whole note head.

1.3.3. Shape of Outer Perimeter

1.3.3.1. The outer perimeter is a slightly skewed ellipse. The left-most point is shifted up from the vertical center slightly (10 units, or ca. 2% of the height), and the right-most point is shifted down from the vertical center by the same amount. The effect on the horizontal axis is to give it a slight negative value.

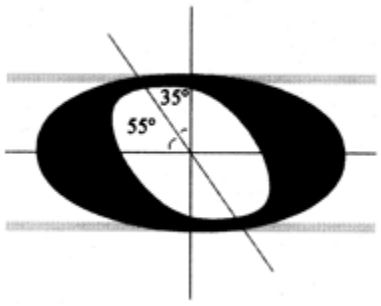
1.3.3.2. The top-most point is shifted to the left of the horizontal center slightly (29 units, or ca. 3% of the width), and the bottom-most point is shifted to the right of the horizontal center by the same amount.

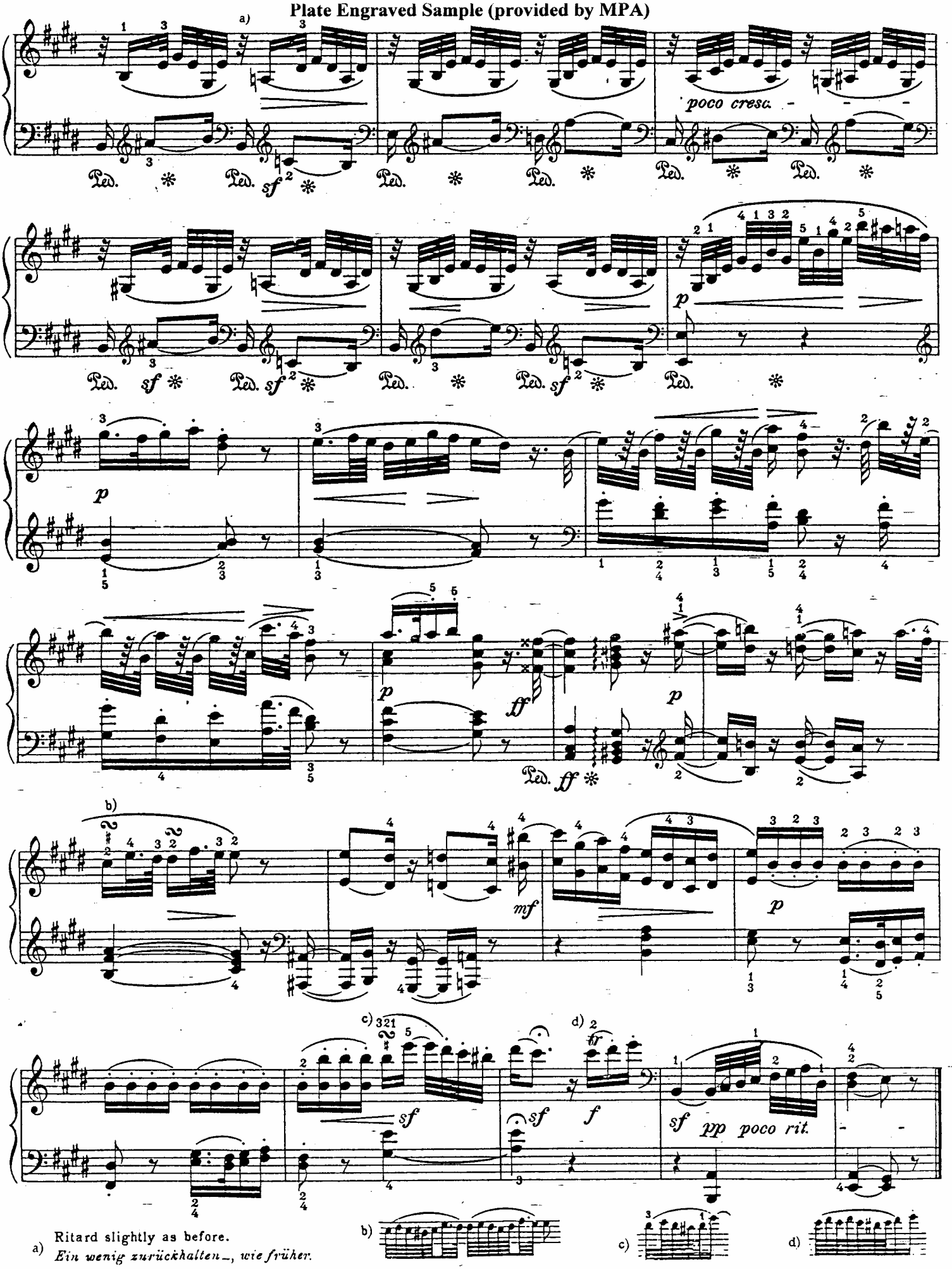
1.3.4. Size and Shape of Counter

1.3.4.1. The counter is most complex of all and will only be dealt with here in general terms.

1.3.4.2. Width/height ratio of counter is 1.19 (514 units wide, 432 units high). The ratio of note width to counter width is thus 1.95 (1002:514) and the ratio of note height to counter height is 1.19 (512:432).

1.3.4.3. The approximate angle of the counter is 35° counterclockwise from the vertical.



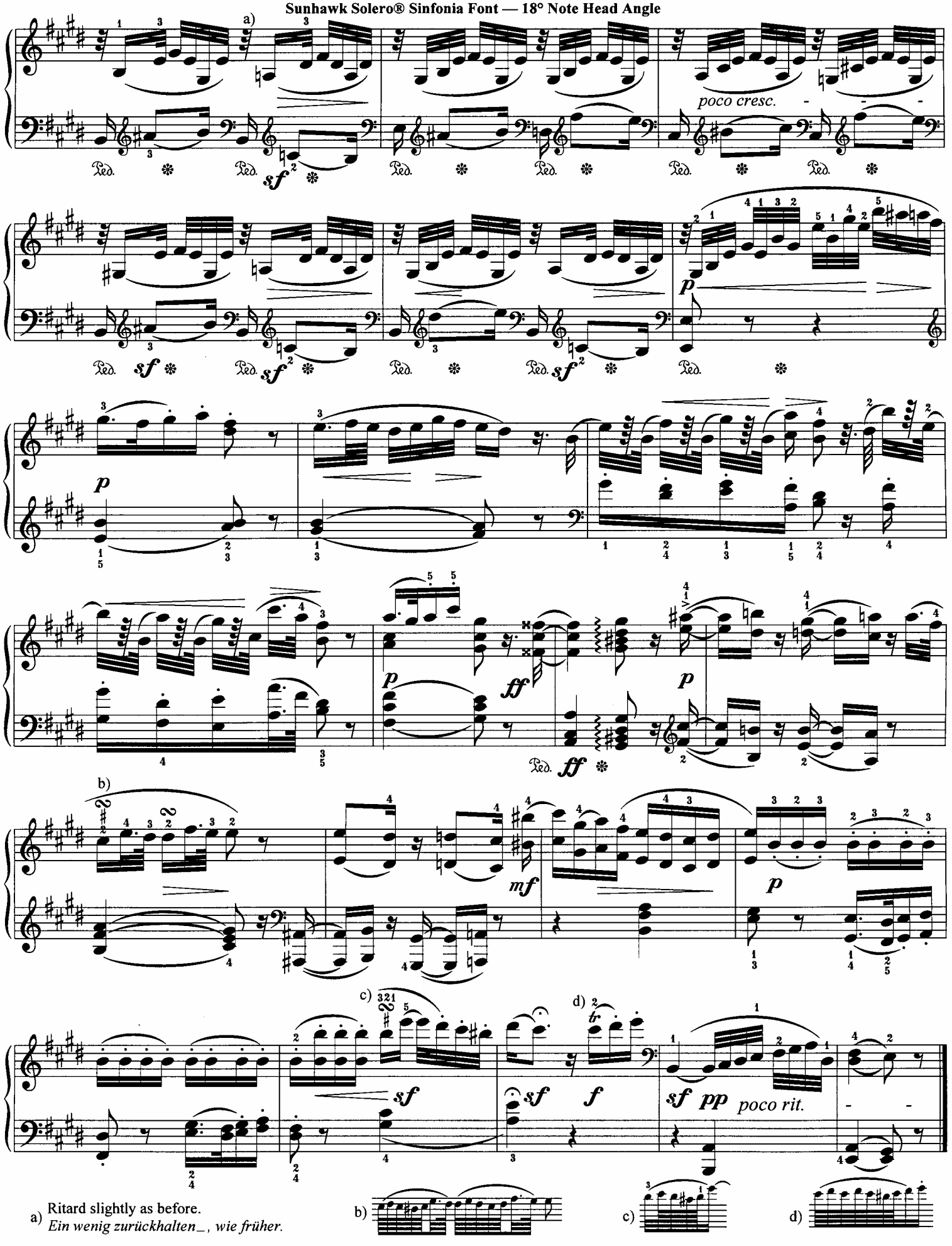


a) Ritard slightly as before.
Ein wenig zurückhalten, wie früher.

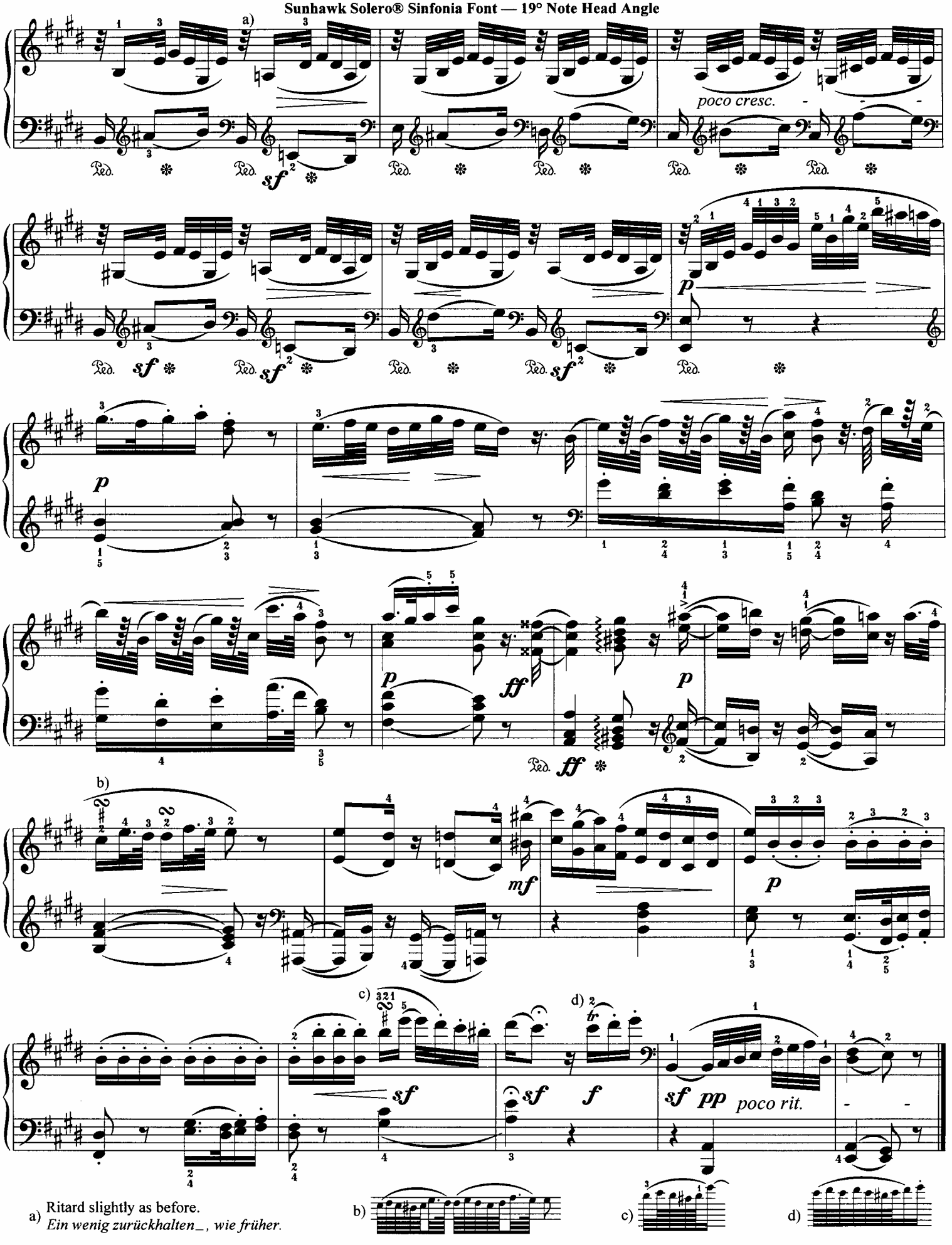
b)

c)

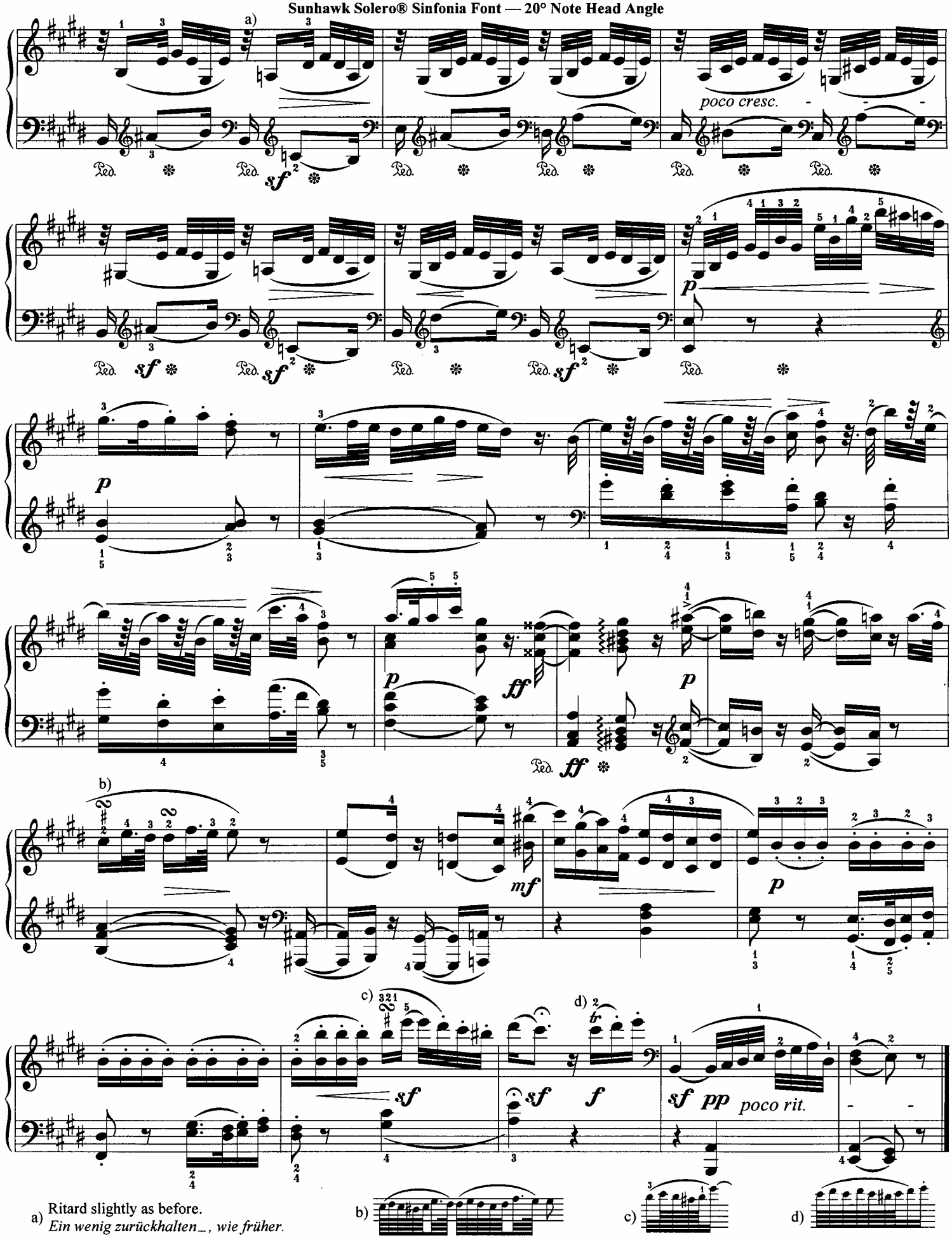
d)



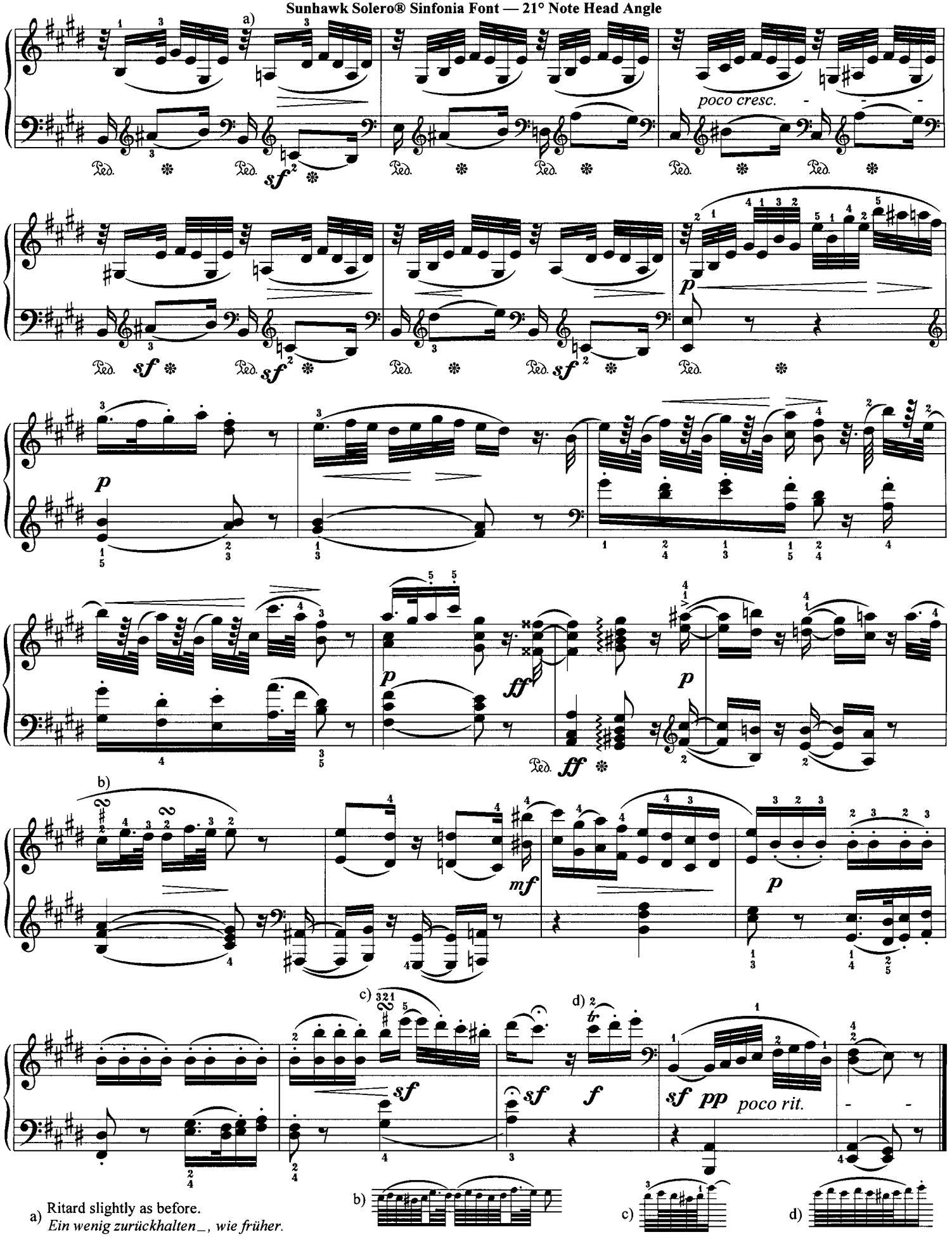
a) Ritard slightly as before.
Ein wenig zurückhalten_, wie früher.



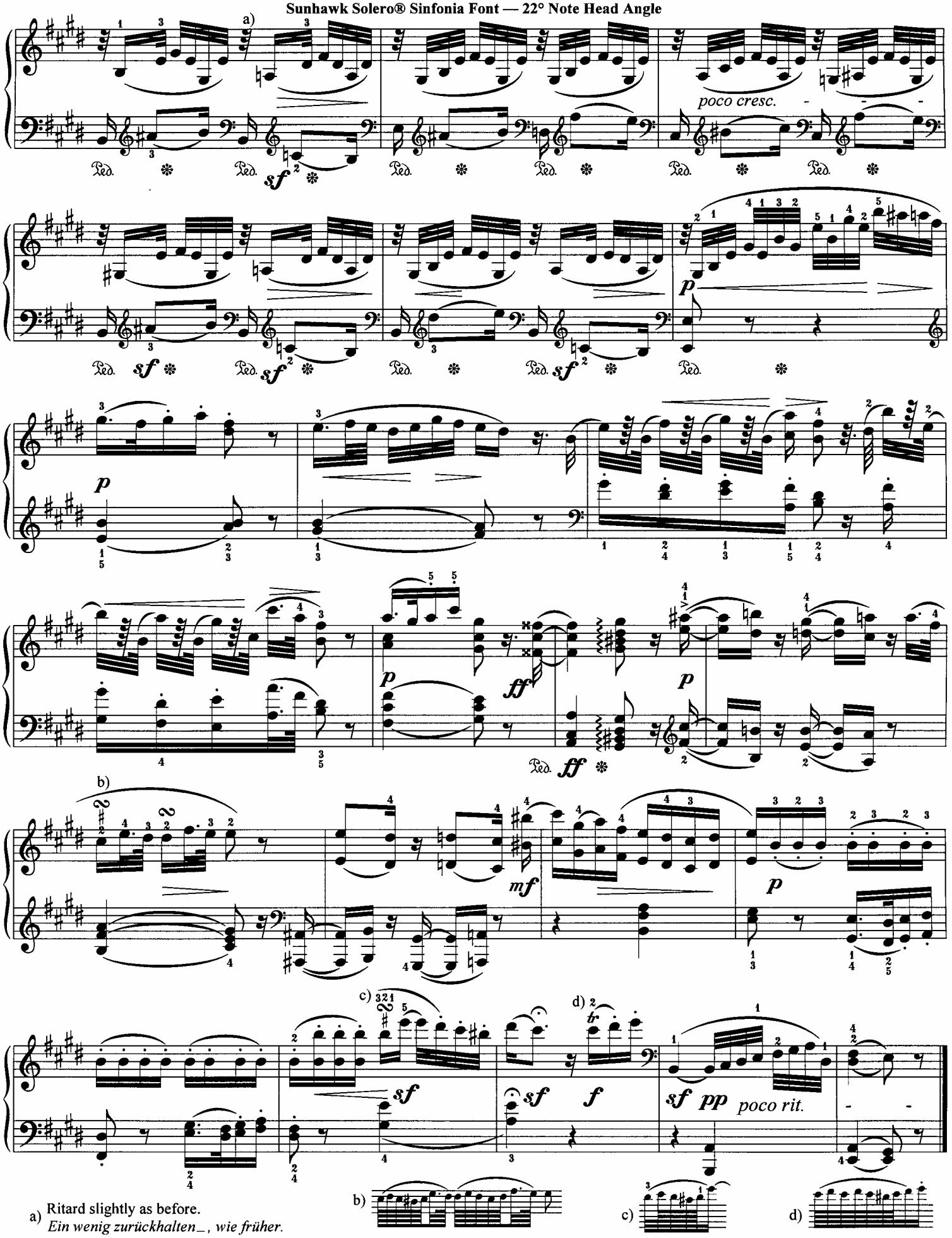
a) Ritard slightly as before.
Ein wenig zurückhalten_, wie früher.



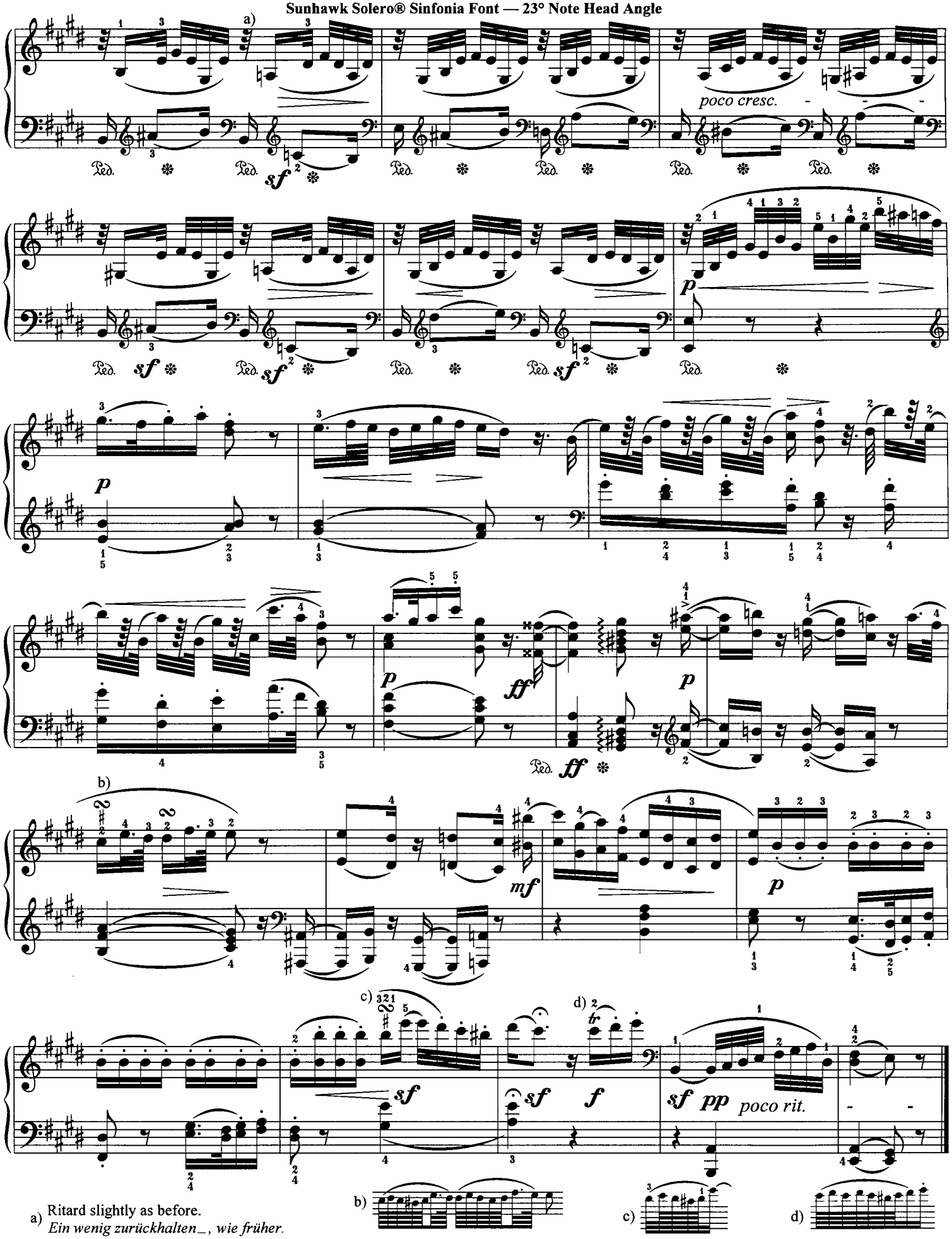
a) Ritard slightly as before.
Ein wenig zurückhalten_, wie früher.

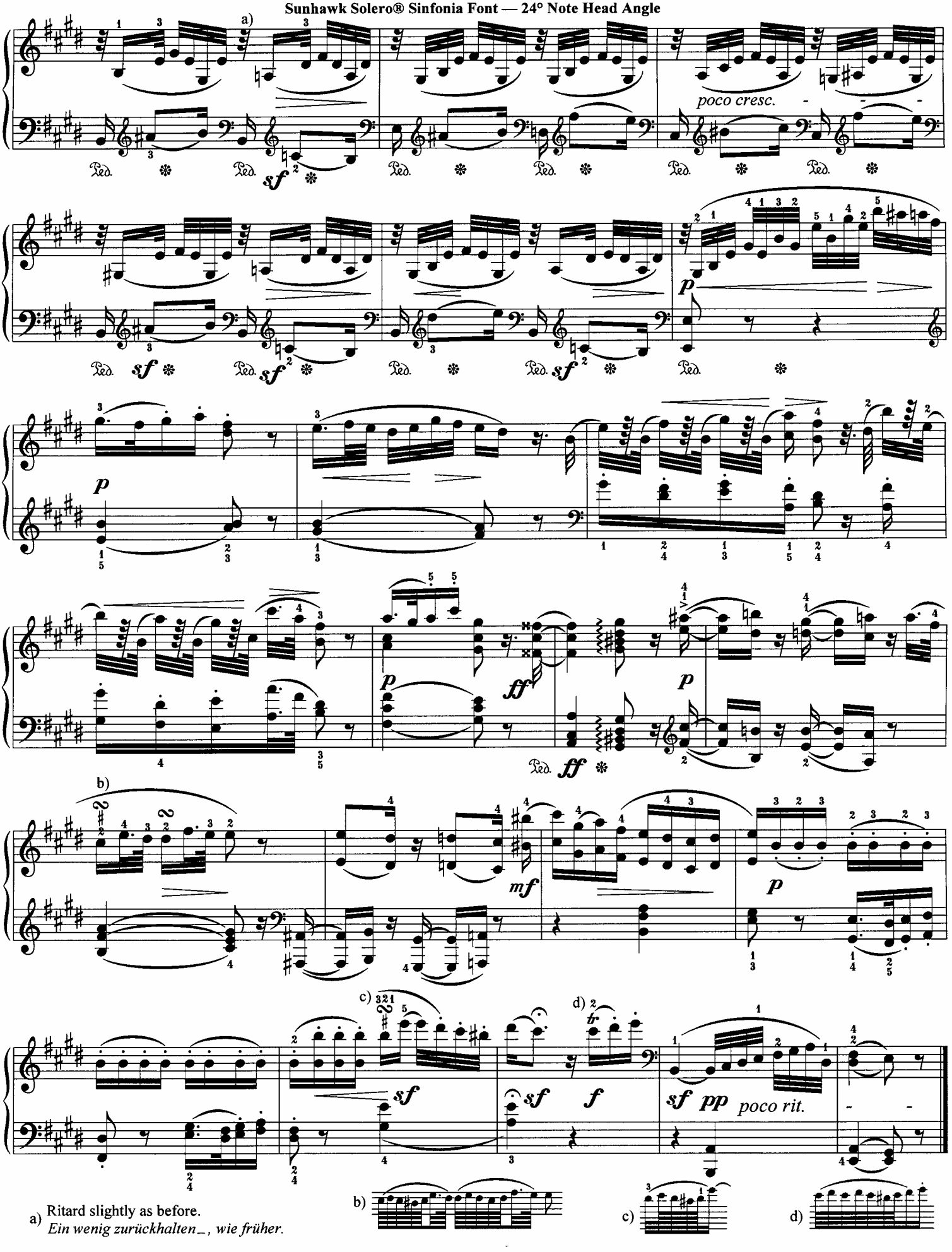


a) Ritard slightly as before.
Ein wenig zurückhalten_, wie früher.

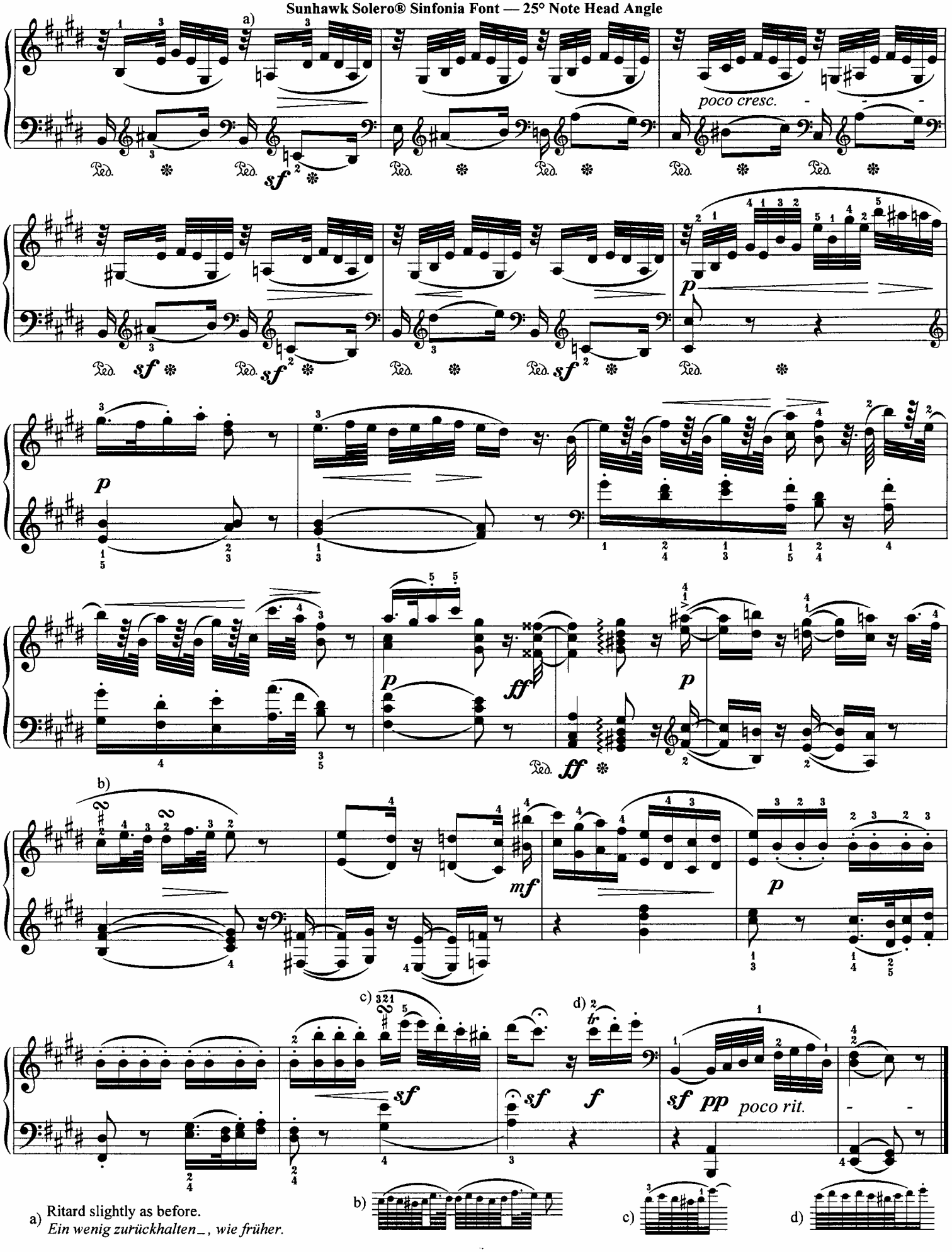


a) Ritard slightly as before.
Ein wenig zurückhalten_, wie früher.


a)
poco cresc.
b)
c)
d)
sf
pp
poco rit.
a) Ritard slightly as before.
Ein wenig zurückhalten_, wie früher.



a) Ritard slightly as before.
Ein wenig zurückhalten_, wie früher.



a) Ritard slightly as before.
Ein wenig zurückhalten_, wie früher.